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Fluid flow generated by switching a magnetic field on or off

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A uniform magnetic field is switched on at time $t = 0$ outside a body of conducting fluid. It is assumed that the field strength increases in time in proportion to $1-e^{-\alpha t}$, where α is a constant of the circuit generating the field. Under the assumption of small magnetic Reynolds number and small magnetic Prandtl number the equations governing the diffusion of the field into the fluid are derived and a simple expression is given for the initial vorticity distribution produced in the fluid. The situation in which an initially uniform field is switched off is also considered. It is shown that, for sufficiently symmetrically shaped bodies of fluid, the vorticity generated by the switching-on of the field is the same as that generated by the switching-off. The particular case of an infinitely long circular cylinder of conducting fluid is considered in detail and an explicit expression is derived for the vorticity distribution.

1. Introduction

This paper generalizes some of the results obtained by Sneyd (1971), who considered the problem of a uniform magnetic field switched on instantaneously across a long circular cylinder of conducting fluid. As the field diffused into the fluid it produced an electric current flow and hence a rotational Lorentz force distribution which set up four eddies – one in each quadrant of the cross-section of the cylinder. It was shown that switching off an initially uniform field produced an identical flow.

It was supposed in that paper that the magnetic field was switched on *instantaneously*. Of course a magnetic field cannot be switched on instantaneously because of the self-inductance of the coils producing the field. If a potential difference V is applied to some electrical circuit the current I in the circuit satisfies the differential equation

$$RI + L\,dI/dt = V,$$

where R is the resistance and L the self-inductance of the circuit. If I is initially zero and a constant potential difference V is applied at time $t = 0$, then

$$I = (V/R)(1-e^{-\alpha t}),$$

where

$$\alpha = R/L. \tag{1.1}$$

The intensity of the magnetic field due to the circuit is proportional to I and its direction is determined by the geometry of the circuit, which is constant in time.

It follows that, if $\mathbf{B}$ is the magnetic field due to a circuit which is switched on at time $t = 0$, then

$$\mathbf{B} = \mathbf{B}_0(1 - e^{-\alpha t}), \tag{1.2}$$

where $\mathbf{B}_0$ is the final steady magnetic field.

If an initially steady current is flowing in the circuit and the potential difference is switched off at time $t = 0$, then

$$I = (V/R)\, e^{-\alpha t}.$$

Thus if the circuit is switched off at time $t = 0$ the magnetic field $\mathbf{B}$ is given by

$$\mathbf{B} = \mathbf{B}_0\, e^{-\alpha t}. \tag{1.3}$$

In this paper, instead of supposing that magnetic fields are switched on and off instantaneously, we shall suppose that they build up or die away according to (1.2) and (1.3).

Section 2 of this paper is concerned with the general situation in which a spatially uniform magnetic field $\mathbf{B}_0$ is switched on outside a body of conducting fluid of arbitrary shape. A simple expression is derived for the initial vorticity distribution generated in the fluid, before it is modified by convection or viscous diffusion. It is shown that, if the body of fluid is an infinitely long cylinder of arbitrary cross-section and $\mathbf{B}_0$ is normal to its generators, or if the body of fluid is symmetric about an axis parallel to $\mathbf{B}_0$, then the vorticity distribution generated in the fluid when the field is switched on is identical to the vorticity distribution generated when an initially uniform field is switched off. This is a generalization of the result proved by Sneyd (1971) for an infinitely long circular cylinder. It is also proved that for less symmetrically shaped bodies of fluid the vorticity produced by the switching-off is not in general the same as that produced by the switching-on.

Section 3 considers the particular problem of a field switched on across a long circular cylinder of fluid. This is the same problem as was considered by Sneyd (1971), but the analysis here makes allowance for a finite switching-on time. An expression is derived for the initial vorticity and it is shown that, in the limit when the switching-on time becomes small in comparison with the time scale for magnetic diffusion, this expression becomes identical with the one derived in the earlier paper.

2. General theory

2.1. *Diffusion of the magnetic field*

A volume V bounded by a surface S is filled with a uniform incompressible fluid of density ρ, kinematic viscosity ν and electrical conductivity σ. l is a typical length scale for V. At time $t = 0$ a spatially uniform magnetic field $\mathbf{B}_0$ is switched on outside this body of fluid, so that far away from the influence of the fluid the magnetic field $\mathbf{B}$ will be given by (1.2). Let $\mathbf{B}_1(\mathbf{x}, t)$ be the magnetic field outside the fluid, and $\mathbf{B}_2(\mathbf{x}, t)$ the magnetic field inside. The region outside the fluid will be assumed non-conducting (e.g. a vacuum) so the electric current flow there will be zero and $\nabla \times \mathbf{B}_1 = 0$. $\mathbf{B}_2$ satisfies the induction equation:

$$\partial \mathbf{B}_2/\partial t = \nabla \times (\mathbf{u} \times \mathbf{B}_2) + \lambda \nabla^2 \mathbf{B}_2,$$

where $\mathbf{u}$ is the fluid velocity, $\lambda = 1/(\mu_0 \sigma)$ and $\mu_0 = 4\pi \times 10^{-7}$ is the magnetic permeability of free space. It will be assumed that the magnetic Reynolds number for the flow inside V is small, i.e. that the flow does not significantly affect the magnetic field. This equation can then be approximated by the simple diffusion equation

$$\partial \mathbf{B}_2/\partial t = \lambda \nabla^2 \mathbf{B}_2.$$

The magnetic field is continuous across S so $\mathbf{B}_1$ and $\mathbf{B}_2$ must be equal on S.

The equations determining $\mathbf{B}_1$ and $\mathbf{B}_2$ can be summarized as follows:

$$\nabla \times \mathbf{B}_1 = 0, \tag{2.1}$$

$$\partial \mathbf{B}_2/\partial t = \lambda \nabla^2 \mathbf{B}_2, \tag{2.2}$$

$$(\mathbf{B}_1)_S = (\mathbf{B}_2)_S, \tag{2.3}$$

$$\mathbf{B}_1 \sim \mathbf{B}_0(1-e^{-\alpha t}) \quad \text{far from } V, \tag{2.4}$$

$$(\mathbf{B}_2)_{t=0} = 0. \tag{2.5}$$

$\tau_0 = l^2/\lambda$ is the time scale for the diffusion of the magnetic field into V and α^{-1} the time scale for the switching-on of the field. Suppose that the field has been left switched on for a time much longer than τ_0 and α^{-1}. The magnetic field will now be everywhere uniform and equal to $\mathbf{B}_0$. If the applied field is then switched off at time $t = 0$, the field far away from the influence of the fluid will decrease according to the equation

$$\mathbf{B} = \mathbf{B}_0 e^{-\alpha t}.$$

Let $\mathbf{B}_1^*$ and $\mathbf{B}_2^*$ be the magnetic field outside and inside the fluid during the switching-off. The equations determining $\mathbf{B}_1^*$ and $\mathbf{B}_2^*$ are

$$\nabla \times \mathbf{B}_1^* = 0, \tag{2.1$'$}$$

$$\partial \mathbf{B}_2^*/\partial t = \lambda \nabla^2 \mathbf{B}_2^*, \tag{2.2$'$}$$

$$(\mathbf{B}_1^*)_S = (\mathbf{B}_2^*)_S, \tag{2.3$'$}$$

$$\mathbf{B}_1^* \sim \mathbf{B}_0 e^{-\alpha t} \quad \text{far from } V, \tag{2.4$'$}$$

$$(\mathbf{B}_2^*)_{t=0} = \mathbf{B}_0. \tag{2.5$'$}$$

When one compares these equations with the equations for $\mathbf{B}_1$ and $\mathbf{B}_2$ it can be seen that the solutions for $\mathbf{B}_1^*$ and $\mathbf{B}_2^*$ are

$$\mathbf{B}_1^* = \mathbf{B}_0 - \mathbf{B}_1 \tag{2.6}$$

and

$$\mathbf{B}_2^* = \mathbf{B}_0 - \mathbf{B}_2. \tag{2.7}$$

2.2. *Vorticity generated by the switching-on and switching-off of the field*

The speed at which the applied magnetic field penetrates the fluid is limited by two factors: the finite magnetic diffusion time τ_0 and the finite switching-on time α^{-1}. The time scale for penetration of the fluid by the field will thus be the maximum of τ_0 and α^{-1}. The ratio of these two time scales is a dimensionless number which we denote by T:

$$T = \tau_0 \alpha = l^2 \alpha \mu_0 \sigma.$$

We shall suppose that T is not very small. If T is very small then the gradual changes in the external field are quickly adjusted by magnetic diffusion so that the field inside the fluid is always approximately equal to the field outside; i.e. it is always approximately uniform. In this case the electric current will be zero and the fluid will not move.

The vorticity generated in the fluid by the rotational Lorentz forces is continuously modified by convection and viscous diffusion. The time scales for these processes are respectively τ_0/R_m and τ_0/P, where R_m is the magnetic Reynolds number and P the magnetic Prandtl number. Since R_m and P are assumed small these time scales will be large compared with τ_0 and hence with α^{-1} (since we assume that α^{-1} is not much larger than τ_0). In calculating the fluid vorticity during the short period in which the magnetic field penetrates the fluid, it is legitimate to neglect the longer-term effects of convection and viscous diffusion. The vorticity equation then becomes simply

$$\partial\boldsymbol{\omega}/\partial t = (1/\rho)\,\nabla\times(\mathbf{j}\times\mathbf{B}_2). \tag{2.8}$$

Suppose that before the magnetic field is switched on the fluid is at rest and $\boldsymbol{\omega} = 0$. The solution of (2.8) is then

$$\boldsymbol{\omega} = \frac{1}{\rho}\int_0^t \nabla\times(\mathbf{j}\times\mathbf{B}_2)\,dt.$$

The total vorticity, $\boldsymbol{\Omega}$ say, generated by the switching-on of the field is given by

$$\boldsymbol{\Omega} = (\boldsymbol{\omega})_{t=\tau_1} = \frac{1}{\rho}\int_0^{\tau_1} \nabla\times(\mathbf{j}\times\mathbf{B}_2)\,dt,$$

where τ_1 is a time scale large compared with τ_0 but small compared with τ_0/R_m or τ_0/P. Assuming that this integral converges as $\tau_1\to\infty$ we may write approximately

$$\boldsymbol{\Omega} = (\boldsymbol{\omega})_{t=\infty} = \frac{1}{\rho}\int_0^{\infty} \nabla\times(\mathbf{j}\times\mathbf{B}_2)\,dt. \tag{2.9}$$

Similarly, the total vorticity $\boldsymbol{\Omega}^*$ generated by the switching-off of an initially uniform field is given by

$$\boldsymbol{\Omega}^* = \frac{1}{\rho}\int_0^{\infty} \nabla\times(\mathbf{j}^*\times\mathbf{B}_2^*)\,dt, \tag{2.10}$$

where $\mathbf{j}^* = (1/\mu_0)\,\nabla\times\mathbf{B}_2^*$ is the electric current density in the fluid during the switching-off. It follows from taking the curl of (2.7) that

$$\mathbf{j}^* = -\mathbf{j}.$$

Substitution in (2.10) gives

$$\boldsymbol{\Omega}^* = \frac{1}{\rho}\int_0^{\infty} \nabla\times(\mathbf{j}\times\mathbf{B}_2)\,dt - \frac{1}{\rho}\int_0^{\infty} \nabla\times(\mathbf{j}\times\mathbf{B}_0)\,dt. \tag{2.11}$$

From (2.11) and (2.9) it follows that

$$\begin{aligned}\boldsymbol{\Omega}^* &= \boldsymbol{\Omega} - \frac{1}{\rho}\int_0^{\infty} (\mathbf{B}_0.\nabla)\,\mathbf{j}\,dt \\ &= \boldsymbol{\Omega} - (\mathbf{B}_0.\nabla)\,\mathbf{f},\end{aligned} \tag{2.12}$$

where
$$\mathbf{f} = \frac{1}{\rho}\int_0^{\infty} \mathbf{j}\,dt.$$

Let us focus attention on the vector $\mathbf{f}$.

$$\nabla.\mathbf{f} = \frac{1}{\rho}\int_0^\infty (\nabla.\mathbf{j})\,dt = 0 \tag{2.13}$$

since in the magnetohydrodynamic approximation $\mathbf{j} = (1/\mu_0)\nabla\times\mathbf{B}$. Now,

$$\begin{aligned}\nabla\times\mathbf{f} &= \frac{1}{\rho}\int_0^\infty (\nabla\times\mathbf{j})\,dt\\ &= \frac{\sigma}{\rho}\int_0^\infty (\nabla\times\mathbf{E})\,dt = -\frac{\sigma}{\rho}\int_0^\infty \frac{\partial\mathbf{B}}{\partial t}\,dt\end{aligned}$$

from Ohm's law and Faraday's law. Thus

$$\nabla\times\mathbf{f} = (-\sigma/\rho)\,[(\mathbf{B})_{t=\infty}-(\mathbf{B})_{t=0}],$$

or

$$\nabla\times\mathbf{f} = (-\sigma/\rho)\,\mathbf{B}_0. \tag{2.14}$$

Finally, since the region outside S is electrically insulating the normal component of $\mathbf{j}$ must be zero on S. It follows that

$$(\mathbf{f}.\mathbf{n})_S = 0, \tag{2.15}$$

where $\mathbf{n}$ is the unit outward normal vector on S.

Equations (2.13)–(2.15) specify the divergence and curl of $\mathbf{f}$ throughout V and the normal component of $\mathbf{f}$ on the boundary S. According to Helmholtz's theorem these three equations then determine $\mathbf{f}$ uniquely. It is possible to write down explicit solutions for $\mathbf{f}$ in the following special cases.

Case (*a*). Suppose that V is an infinitely long cylinder of uniform cross-section and that $\mathbf{B}_0$ is normal to its generators. Choose a Cartesian co-ordinate system with the z axis parallel to the generators of the cylinder and the x axis parallel to $\mathbf{B}_0$. Then the vector $-B_0(\sigma/\rho)\,y\mathbf{k}$ will satisfy (2.13)–(2.15), so, by Helmholtz's uniqueness theorem,

$$\mathbf{f} = -B_0(\sigma/\rho)\,y\mathbf{k}.$$

Notice in this case that $(\mathbf{B}_0.\nabla)\mathbf{f} = 0$ so according to (2.12) $\mathbf{\Omega} = \mathbf{\Omega}^*$. In this case the vorticity generated by switching off the field is equal to that generated by switching it on.

Case (*b*). Suppose that V is symmetric about an axis parallel to $\mathbf{B}_0$ (e.g. a sphere). Choose a cylindrical polar co-ordinate system (r,θ,z) so that the axis of symmetry coincides with the z axis. Then the vector $-\frac{1}{2}B_0(\sigma/\rho)\,r\,\mathbf{e}_\theta$ satisfies (2.13)–(2.15) so

$$\mathbf{f} = -\tfrac{1}{2}B_0(\sigma/\rho)\,r\mathbf{e}_\theta. \tag{2.16}$$

Again it follows that $(\mathbf{B}_0.\nabla)\,\mathbf{f} = 0$ and that $\mathbf{\Omega} = \mathbf{\Omega}^*$.

This result that $\mathbf{\Omega} = \mathbf{\Omega}^*$ in case (*a*) and case (*b*) does not depend on the exponential build-up and decay of the applied magnetic field. So long as $B \propto f(t)$ and $B^* \propto 1-f(t)$, where $f(t)$ is any continuous function such that $f(0) = 0$ and $f(\infty) = 1$, it will still be true that $\mathbf{\Omega} = \mathbf{\Omega}^*$.

It is not in general true that $\mathbf{\Omega} = \mathbf{\Omega}^*$ for less symmetrically shaped bodies of fluid. For suppose it is true that $\mathbf{\Omega} = \mathbf{\Omega}^*$, i.e. that

$$(\mathbf{B}_0.\nabla)\,\mathbf{f} = 0. \tag{2.17}$$

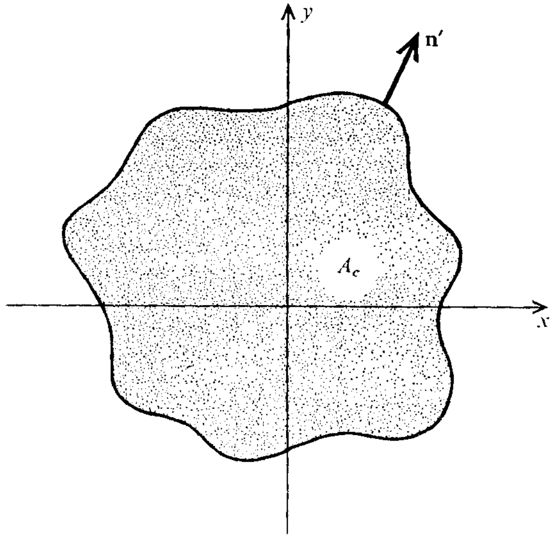

FIGURE 1

If the z axis is chosen parallel to $\mathbf{B}_0$ one finds that

$$\partial \mathbf{f}/\partial z = 0.$$

This equation, together with (2.14), implies that

$$\partial f_z/\partial x = \partial f_z/\partial y = 0$$

and hence that f_z is constant. Provided that the surface of the body of fluid is smooth there must be at least one point on the surface where the tangent plane is normal to the z axis, i.e. where $\mathbf{n} = \mathbf{k}$. At this point (2.15) shows that $f_z = 0$ and since f_z is a constant we must have $f_z = 0$ everywhere. Thus the vector $\mathbf{f}$ must be of the form

$$\mathbf{f} = f_x(x,y)\,\mathbf{i} + f_y(x,y)\,\mathbf{j}.$$

Let A_c be the surface formed by the intersection of the plane $z = c$ (a constant) with the body of fluid (see figure 1). The problem of determining $\mathbf{f}$ now reduces to a two-dimensional problem in the region A_c. Equations (2.13)–(2.15) show that in A_c we must have

$$\left.\begin{aligned} \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} = 0, \quad \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} = (-\sigma/\rho)\,B_0, \\ (\mathbf{f}\,.\,\mathbf{n}')_S = 0, \end{aligned}\right\} \tag{2.18}$$

where $\mathbf{n}'$ is a unit vector in the x, y plane and normal to the boundary of A_c. The two-dimensional form of Helmholtz's theorem shows that the equations (2.18) determine f_x and f_y and hence $\mathbf{f}$ uniquely.

Thus if it is true that $\mathbf{\Omega} = \mathbf{\Omega}^*$ then $\mathbf{f}$ is uniquely determined by the shape of *any one particular* cross-section A_c of the body of fluid. But if $\mathbf{f}$ is fixed by the shape of one cross-section A_c there is no guarantee that the condition $(\mathbf{f}\,.\,\mathbf{n})_S = 0$ will not be violated on another cross-section $A_{c'}$. For example, suppose that

A_c is a circle. Then $\mathbf{f}$ is given by (2.16). Now the condition $(\mathbf{f}.\mathbf{n})_S = 0$ will be violated at some point on S unless all the other cross-sections are concentric circles.

3. Field switched on across a circular cylinder

3.1. *Diffusion of the field*

This section is concerned with the particular case of an infinitely long circular cylinder of conducting fluid and a uniform applied magnetic field $\mathbf{B}_0$ normal to its axis. This problem was considered by Sneyd (1971), but in that paper it was assumed that the external magnetic field was switched on instantaneously, i.e. that $T = \infty$. Here the same problem will be solved for finite values of T.

A cylindrical polar co-ordinate system (r, θ, z) is used so that the equation of the fluid cylinder is $r \leqslant a$. The x axis is chosen parallel to the applied magnetic field. The magnetic fields $\mathbf{B}_1$ and $\mathbf{B}_2$ are represented by their corresponding stream functions ψ_1 and ψ_2:

$$\mathbf{B}_i = \frac{1}{r}\frac{\partial \psi_i}{\partial \theta}\mathbf{e}_r - \frac{\partial \psi_i}{\partial r}\mathbf{e}_\theta \quad (i = 1, 2).$$

Equations (2.1)–(2.5) when written in terms of ψ_1 and ψ_2 become

$$\nabla^2 \psi_1 = 0, \tag{3.1}$$

$$\partial \psi_2 / \partial t = \lambda \nabla^2 \psi_2, \tag{3.2}$$

$$\left(\frac{\partial \psi_1}{\partial r}\right)_{r=a} = \left(\frac{\partial \psi_2}{\partial r}\right)_{r=a}, \quad \left(\frac{\partial \psi_1}{\partial \theta}\right)_{r=a} = \left(\frac{\partial \psi_2}{\partial \theta}\right)_{r=a}, \tag{3.3}$$

$$\psi_1 \sim B_0 r \sin\theta (1 - e^{-\alpha t}) \quad \text{as} \quad r \to \infty, \tag{3.4}$$

$$(\psi_2)_{t=0} = 0. \tag{3.5}$$

These equations may be solved by writing

$$\psi_1 = f_1(r, t)\sin\theta, \quad \psi_2 = f_2(r, t)\sin\theta$$

and taking the Laplace transform of (3.2) in t. When the Laplace transform of $f_2(r, t)$ is inverted by contour integration one obtains the following solution for ψ_2:

$$\psi_2 = B_0 a \sin\theta\, \xi - \frac{2B_0 a \sin\theta J_1(T\xi)\, e^{-\alpha t}}{T J_0(T)} - 4B_0 a \sin\theta \sum_{n=1}^{\infty} \frac{J_1(\lambda_n \xi)\, e^{-\lambda \lambda_n^2 t/a^2}}{\lambda_n^2 (1 - \lambda_n^2/T^2) J_1(\lambda_n)}, \tag{3.6}$$

where $\xi = r/a$, the λ_n are the positive zeros of $J_0(x)$ and $T = a^2\alpha/\lambda$. It can be seen that as $T \to \infty$ equation (3.6) tends to the corresponding equation, equation (2.5), in Sneyd (1971). If T is small

$$J_1(T\xi)/T J_0(T) \sim \xi$$

and

$$\psi_2(r, \theta, t) \sim B_0 r \sin\theta (1 - e^{-\alpha t}),$$

which is just the stream function for the applied magnetic field at time t.

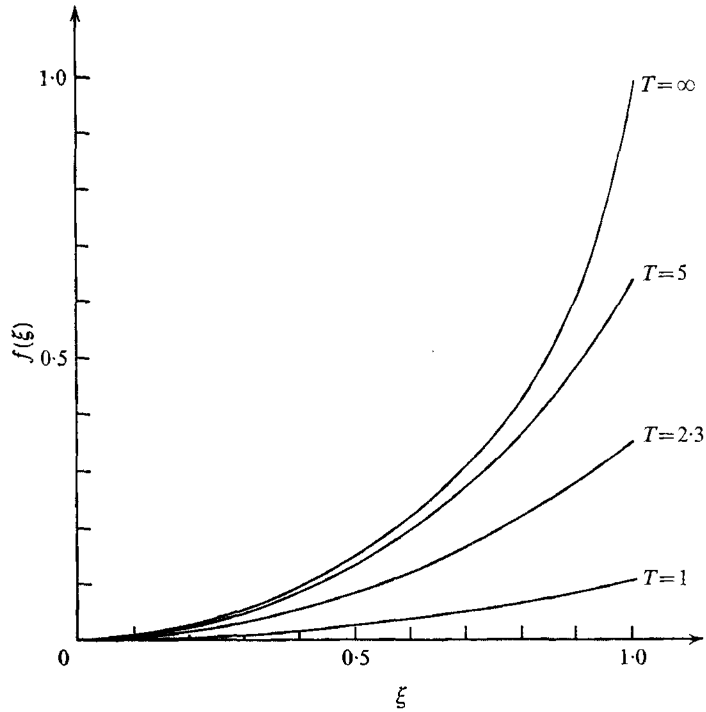

FIGURE 2

It might appear from (3.6) that ψ_2 has a singularity as $T \to \lambda_n$, but this is not so since the singularity of the term in the infinite series is cancelled by a corresponding singularity in the second term of (3.6).

3.2. *Fluid motion generated by the rotational Lorentz force*

The current density $\mathbf{j}$ induced in the cylinder of fluid can now be calculated:

$$\mathbf{j} = (1/\mu_0)\nabla \times \mathbf{B}_2 = -(1/\mu_0)\nabla^2\psi_2\mathbf{k}.$$

The total vorticity $\mathbf{\Omega}$ generated by the switching-on of the field is given by (2.9):

$$\mathbf{\Omega} = \frac{\mathbf{k}}{\rho\mu_0 r}\int_0^\infty \frac{\partial(\psi_2, \nabla^2\psi_2)}{\partial(r,\theta)}\,dt.$$

Using the formula for ψ_2 given by (3.6) one finds (after a little algebra and summation of series by contour integration) that

$$\mathbf{\Omega} = (B_0^2 \sin 2\theta/\rho\mu_0\lambda) f(\xi)\,\mathbf{k},$$

where $$f(\xi) = \frac{2J_2(T\xi)\,I_1(T\xi)}{\xi T J_0(T)\,I_0(T)} + \frac{8}{\xi}\sum_{n=1}^{\infty}\frac{J_2(\lambda_n\xi)\,I_1(\lambda_n\xi)}{\lambda_n^2(1-\lambda_n^4/T^4)\,J_1(\lambda_n)\,I_0(\lambda_n)}.$$

Figure 2 shows graphs of the function $f(\xi)$ for various values of T. It can be seen that at any given point in the fluid the vorticity generated decreases as T decreases.

The initial flow produced by the switching-on of the field is two-dimensional and incompressible so there exists a stream function χ such that

$$\mathbf{u} = \frac{1}{r}\frac{\partial \chi}{\partial \theta}\,\mathbf{e}_r - \frac{\partial \chi}{\partial r}\,\mathbf{e}_\theta.$$

Now, $$\nabla^2 \chi = -\omega_z = -(B_0^2 \sin 2\theta/\rho\mu_0\lambda) f(\xi).$$

Let $$\chi = -g(\xi)\sin 2\theta.$$

Then the above equation becomes

$$g'' + \frac{g'}{\xi} - \frac{4g}{\xi^2} = (B_0^2 a^2/\rho\mu_0\lambda) f(\xi).$$

The boundary conditions on g are that it must remain finite at $\xi = 0$ and that $g(1) = 0$, i.e. that the surface of the cylinder must be a streamline. The Green's function $G(\xi, t)$ for the problem

$$g'' + g'/\xi - 4g/\xi^2 = 0, \quad g(0) < \infty, \quad g(1) = 0$$

is given by

$$G(\xi, t) = \begin{cases} \frac{1}{4}(t^3 - 1/t)\,\xi^2, & 0 \leqslant \xi < t, \\ \frac{1}{4}(\xi^2 - 1/\xi^2)\,t^3, & t < \xi \leqslant 1. \end{cases}$$

$$\chi = -\frac{B_0^2 a^2 \sin 2\theta}{4\rho\mu_0\lambda}\left\{\left(\xi^2 - \frac{1}{\xi^2}\right)\int_0^\xi t^3 f(t)\,dt + \xi^2 \int_\xi^1 (t^3 - 1/t) f(t)\,dt\right\}.$$

4. Conclusions

The graphs in figure 2 show that the finite switching-on time of the applied magnetic field does not significantly affect the overall pattern of the flow produced. The most important effect of the finite switching-on time is on the flow *speeds*: for large T the flow is rapid, and for small T it is slow.

It is difficult to estimate a likely experimental value for T for any particular body of conducting fluid. The value of T depends on the large number of factors involved in the design of the magnet generating the applied field: the number of turns of wire, the diameter and material of the wire, etc. It should be possible to design magnets to make T large or small, so it should be possible to verify experimentally the conclusions of § 3.

REFERENCE

Sneyd, A. D. 1971 Generation of fluid motion in a circular cylinder by an unsteady applied magnetic field. *J. Fluid Mech.* **49**, 817.